1. Derive a finite difference approximation for  in terms of ,  and . What is the order of accuracy of this approximation?

2. The location of an object at various times was measured as follows:

|  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Time (min) | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| Distance (cm) | 0 | 3 | 14 | 39 | 84 | 155 | 258 | 399 | 584 | 819 |

Estimate the speed and acceleration of the object at 5 minutes using (a) Forward difference, O(h2); (b) Backward difference, O(h2); (c) Central difference, O(h2); and (d) Richardson extrapolation, O(h6) using three central differences of O(h2). (e) Perform an error analysis for the results obtained in the above problem, using the fact that the distance is given by x = t + t2 + t3.

3. The flow rate through a circular pipe is given by , where *v* is the velocity at a distance of *r* from the centre of pipe and *r0* is the radius of the pipe. If the velocity is approximated by (in m/s), and the pipe radius is 12 cm, compute Q using (a) trapezoidal rule with h = 2 cm; (b) Simpson’s one-third rule with h = 3 cm; (c) Simpson’s three-eighth rule with h = 4 cm; and (d) 3-point Gauss-Legendre quadrature. (e) Perform an error analysis for the results obtained in the above problem, using the true value of the flow rate as 0.0738902 m3/s.